ToulBar2 to solve Weighted Partial Max-SAT[*]

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Abstract

We used a standard Depth First Branch and Bound (DFBB) for the Eighth Max-SAT Evaluation. Our solver combines a limited discrepancy search with a maximum discrepancy of 1 with a complete DFBB algorithm. It exploits soft local consistencies derived bounds and a restricted form of soft neighborhood substitutability called dead-end elimination. Weighted Partial Max-SAT are directly translated into Cost Function Networks with Boolean domains.

Cost Function Networks

Cost Function Networks (CFNs), also known as weighted Constraint Satisfaction problems [Meseguer et al., 2006] is a mathematical model which has been derived from constraint satisfaction problems by replacing constraints with cost functions. In a CFN, we are given a set of variables with an associated finite domain and a set of local cost functions. Each cost function involves some variables and associates a non negative integer cost to each of the possible combinations of values they may take. The usual problem considered is to assign all variables in a way that minimizes the sum of all costs. Given an initial upper bound U, all costs above U can be considered as infinite, enabling the expression of usual constraints (strictly forbidden tuples). The first presentation of such "relaxed" constraint networks was given by [Rosenfeld et al., 1976] using max/min and max/* operations. It was later generalized to algebraic frameworks in [Schiex et al., 1995; Bistarelli et al., 1997]. Such frameworks allow for the concise description of global cost distributions, defined by the combination (usually the sum) of local cost functions.

Weighted Partial Max-SAT are directly translated into CFN with initial upper bound U set to *top* as defined by the *wcnf* format or by the sum of the clause weights plus one if *top* was not available. Multiple binary or ternary clauses on the same set of variables are merged into a single cost function.

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Toulbar2 solver

Toulbar2 is an exact solver for CFNs. As a default, it uses a Depth First Branch and Bound algorithm to identify a minimum cost assignment and prove its optimality. The lower bound used for pruning during tree search is based on "soft local consistency enforcing" [Cooper et al., 2010]. Soft local consistency allows to transform a CFN into an equivalent CFN with an associated non naive constant cost function that provides a strong incremental lower bound. The default level of enforcing used in ToulBar2 is known as "Existential Directed Arc Consistency" or EDAC [Larrosa et al., 2005]. This processing is only applied to cost function of arity 3 or less. Higher arity clauses are delayed until they are always satisfied (and can be removed) or their arity has been reduced due to variable assigments.

The tree explored is a binary tree where each node corresponds to either fixing a chosen Boolean variable to false or true. Beyond this, ToulBar2 integrates on-the-fly variable elimination [Larrosa, 2000] which eliminates any variable with a low degree (default 3) during search.

To choose a variable and a value at each node, dedicated heuristics are used. The variable chosen is selected using weighted degree [Boussemart et al., 2004] The value selected is a value with minimum cost in the associated unary cost function involving just this variable. This cost function can also be non naive, following local consistency enforcing, allowing to direct the search to promising area rapidly.

We add a dominance rule called Dead-End Elimination (DEE), originally studied in computational biology [Desmet et al., 1992; Goldstein, 1994], which allows more pruning during the search. DEE is similar to soft neighborhood substitutability [Lecoutre et al., 2012]. The idea is to automatically detect values in the domain of a variable which are *dominated* by another *dominant* value of the domain such that any solution using the dominant value instead of the dominated ones has a better score. Dominated values can be safely removed from the domain of the variable.

For the Max-SAT Evaluation, we used options "-b: -l=1 -dee=1".

Conclusion

ToulBar2 includes other facilities which cannot be described here, including stronger preprocessing algorithms, and the ability to express global cost functions (involving a large number of variables, with a fixed semantics and dedicated efficient local consistency algorithms). ToulBar2 has been used to solve real problems in resource allocation [Cabon et al., 1999], pedigree analysis [Sanchez et al., 2008] and bioinformatics [Zytnicki et al., 2008; Allouche et al., 2012].

People interested in using or contributing to ToulBar2 can go the software forge hosting the project at https://mulcyber.toulouse.inra.fr/projects/toulbar2. More information on ToulBar2, including a Web service for running ToulBar2 and a large collection of benchmarks (real, academic and random) of cost function networks can be found at https://costfunction.org.

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